ENHANCED MULTI-MASS MODEL IN RSCAD

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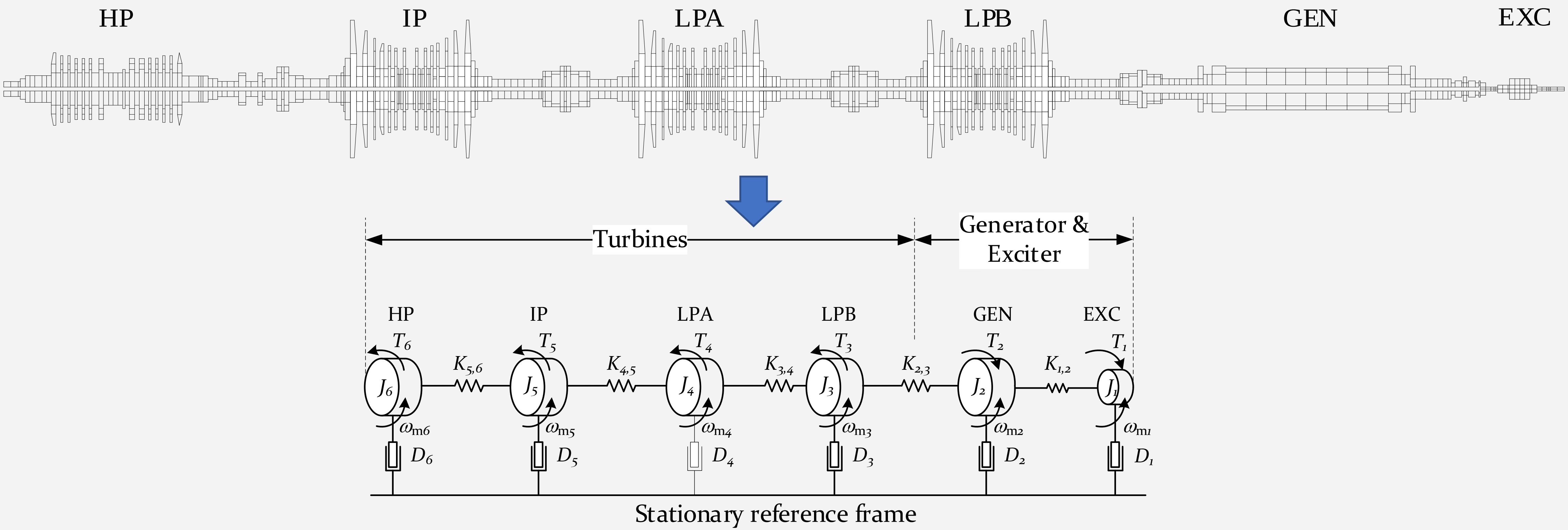
INTRODUCTION

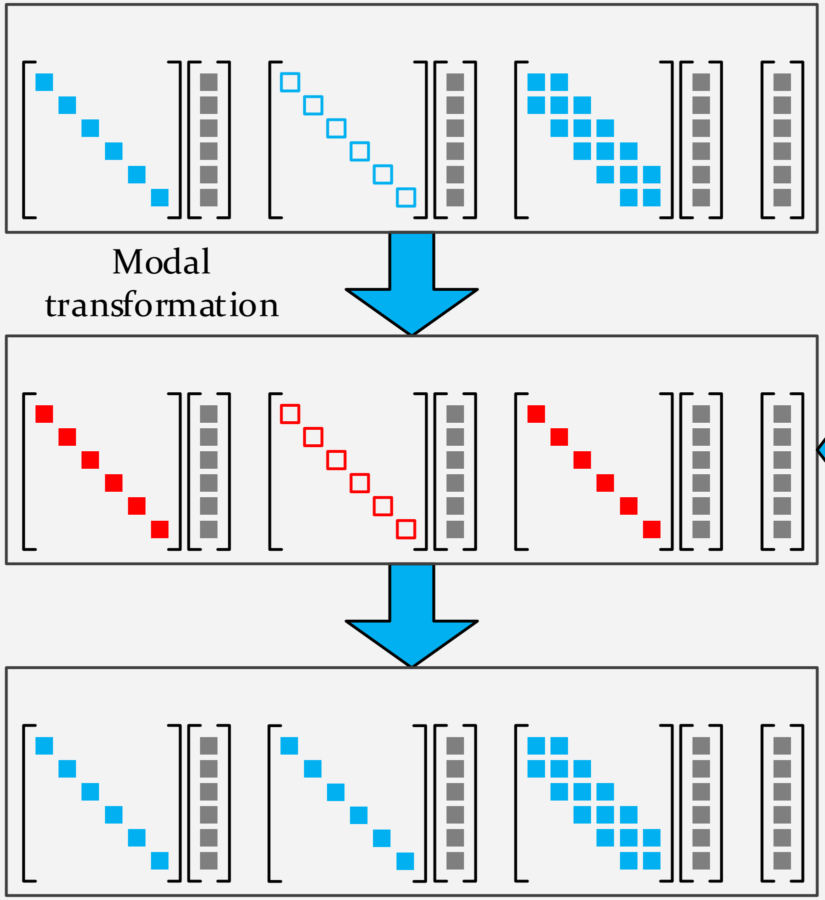


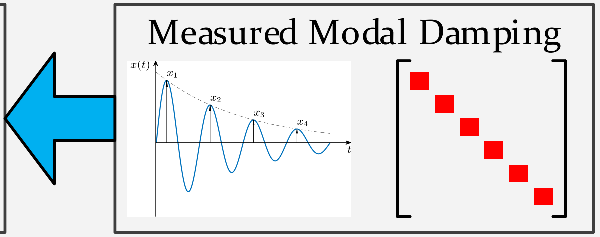
The differential equation describing the dynamic of a mass that is connected by an elastic shaft to its adjacent masses:

$$2H_i\dot{\omega}_{\mathrm{m},i}^{\mathrm{pu}} = T_{\mathrm{m},i}^{\mathrm{pu}} - D_i^{\mathrm{pu}}\omega_{\mathrm{m},i}^{\mathrm{pu}} - K_{i,i-1}^{\mathrm{pu}}\left(\theta_i^{\mathrm{pu}} - \theta_{i-1}^{\mathrm{pu}}\right) - K_{i,i+1}^{\mathrm{pu}}\left(\theta_i^{\mathrm{pu}} + \theta_{i-1}^{\mathrm{pu}}\right)$$

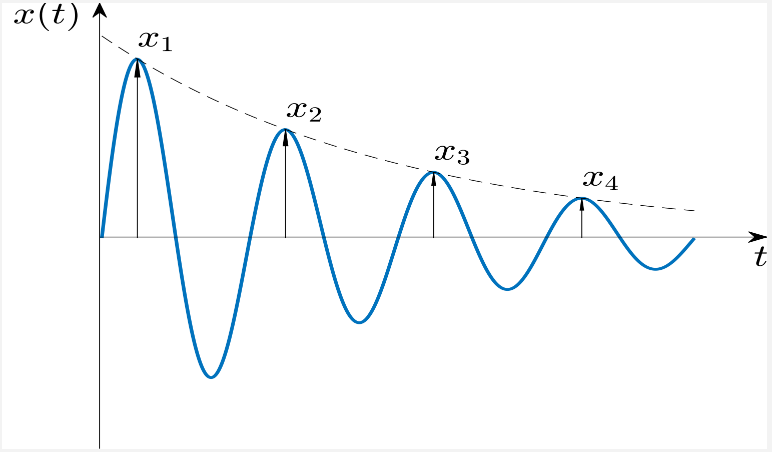
SOLUTION: ADDING MODAL DAMPING OBTAINED FROM MEASUREMENTS

$$\mathbf{H}\ddot{\boldsymbol{\theta}} + \mathbf{D}\dot{\boldsymbol{\theta}} + \mathbf{K}\boldsymbol{\theta} = \mathbf{T}$$

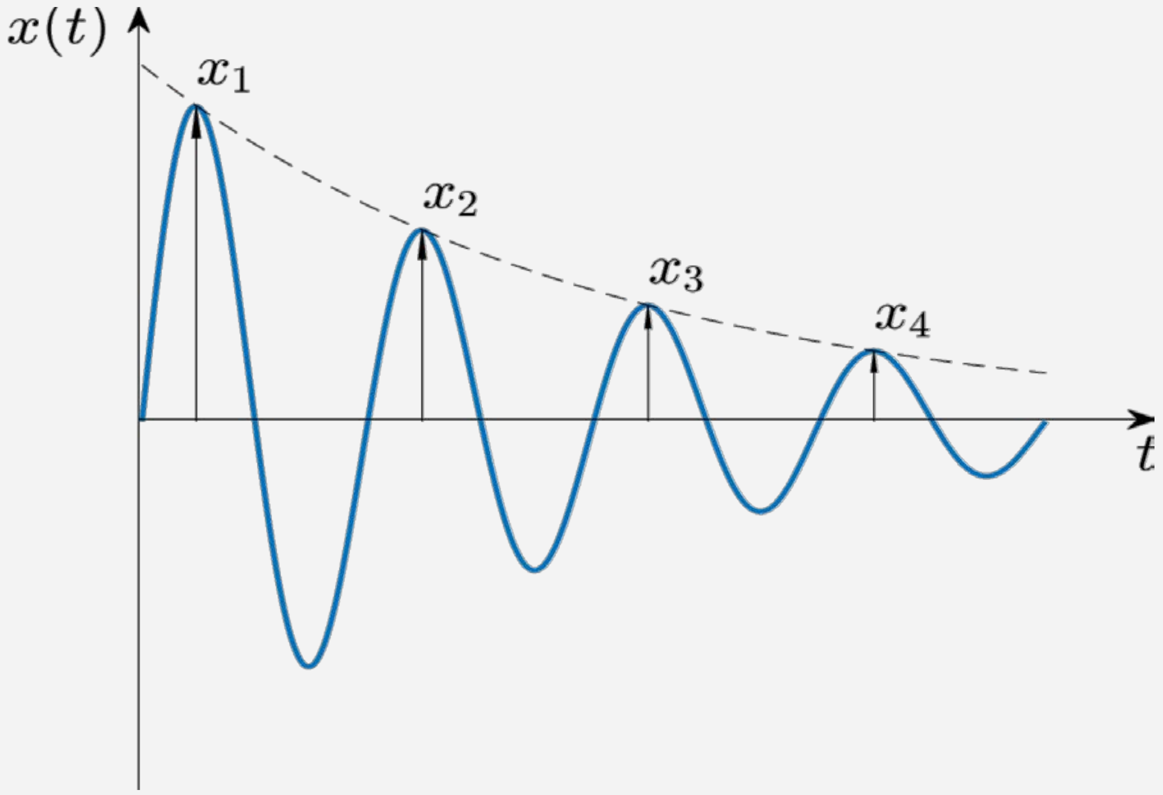




Modal damping as logarithmic decrement



LOGARITHMIC DECREMENT



Solution of the equation of motion:

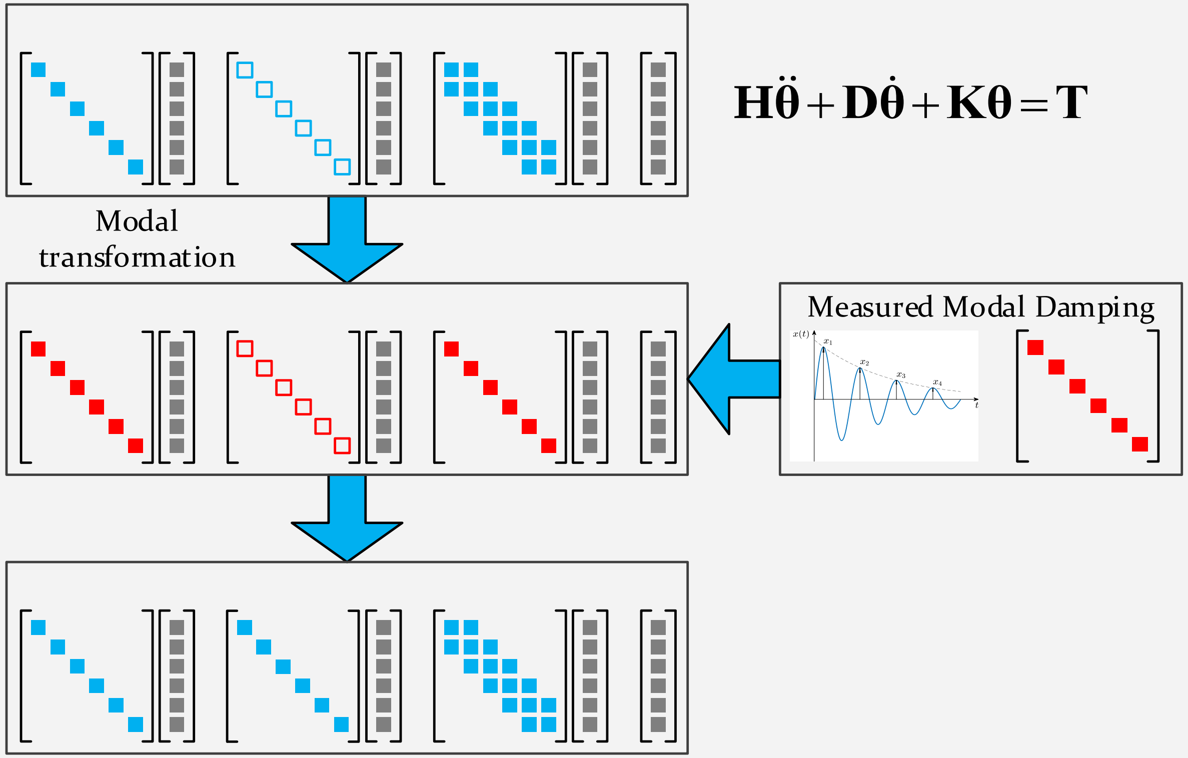
$$x(t) = Ae^{-\zeta\omega_n t}\cos(\omega_d t - \Psi)$$

$$\frac{x(t)}{x(t+nT)} = \frac{Ae^{-\zeta\omega_n t}\cos(\omega_d t - \Psi)}{Ae^{-\zeta\omega_n(t+\mathrm{nT})}\cos(\omega_d(t+\mathrm{nT}) - \Psi)} = e^{\zeta\omega_n \mathrm{nT}}$$

$$\Lambda = \frac{1}{n}\ln\left[\frac{x(t)}{x(t+nT)}\right] = \zeta\omega_n T$$

$$D_\mathrm{m} = 2\cdot f\cdot J\cdot\Lambda$$

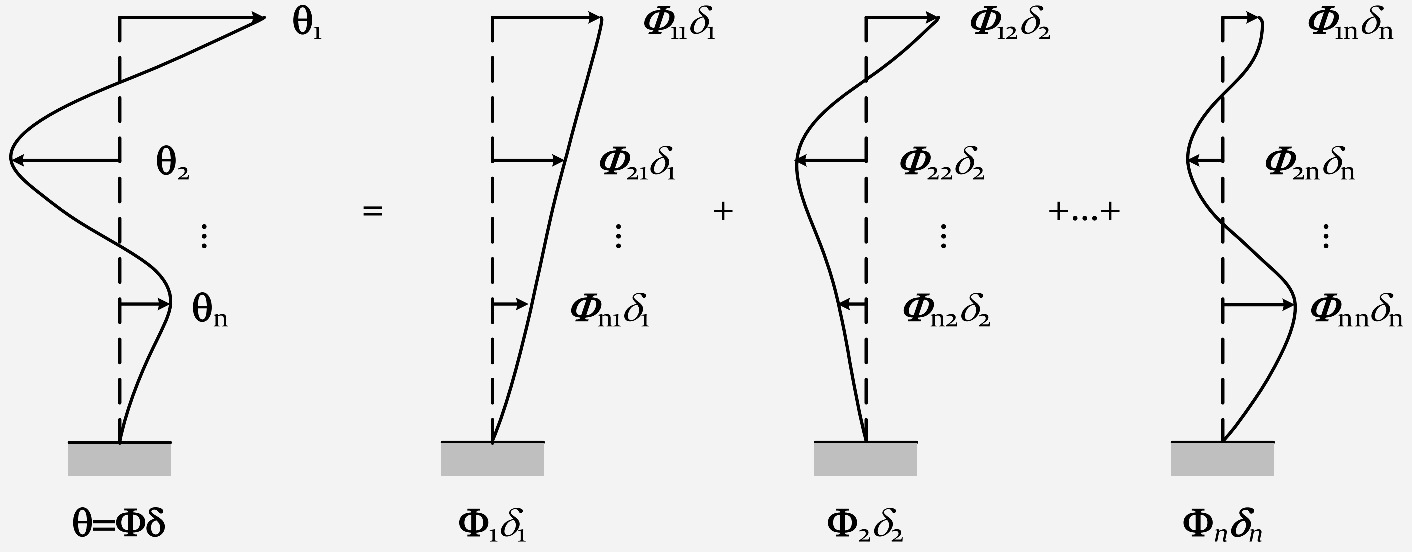
LOGARITHMIC DECREMENT



$$\mathbf{H}\ddot{\boldsymbol{\theta}} + \mathbf{D}\dot{\boldsymbol{\theta}} + \mathbf{K}\boldsymbol{\theta} = \mathbf{T}$$

$$D_{\mathrm{m}} = 2 \cdot f \cdot J \cdot \Lambda$$

MODAL ANALYSIS OF THE MECHANICAL SYSTEM



Superposition of the modal contribution:

$$\boldsymbol{\theta} = \sum_{i=1}^{n} \mathbf{X_i}\boldsymbol{\delta_i} = \mathbf{X}\boldsymbol{\delta}$$

Construction of the transformation matrix:

$$\mathbf{H}\ddot{\boldsymbol{\delta}}\mathbf{X} + \mathbf{K}\boldsymbol{\delta}\mathbf{X} = 0$$

Eigenvalue problem, assuming $\boldsymbol{\delta}(t) = \hat{\boldsymbol{\delta}}e^{\boldsymbol{\Lambda}t}$:

$$(\boldsymbol{\Lambda}^2\mathbf{H} + \mathbf{K})\mathbf{X} = 0 \qquad \boldsymbol{\Lambda} = diag(\lambda_1, \lambda_2 \ldots \lambda_n)$$

Real system

$$\mathbf{H}\ddot{\boldsymbol{\theta}} + \mathbf{D}\dot{\boldsymbol{\theta}} + \mathbf{K}\boldsymbol{\theta} = \mathbf{T}$$

Transformation

$$\underbrace{\left(\mathbf{X}^T\mathbf{H}\mathbf{X}\right)}_{\mathbf{H_m}}\ddot{\boldsymbol{\delta}} + \underbrace{\left(\mathbf{X}^T\mathbf{D}\mathbf{X}\right)}_{\mathbf{D_m}}\dot{\boldsymbol{\delta}} + \underbrace{\left(\mathbf{X}^T\mathbf{K}\mathbf{X}\right)}_{\mathbf{K_m}}\boldsymbol{\delta} = \underbrace{\mathbf{X}^T\mathbf{T}}_{\mathbf{T_m}}$$

Modal system

$$\mathbf{H_m}\ddot{\boldsymbol{\delta}} + \mathbf{D_m}\dot{\boldsymbol{\delta}} + \mathbf{K_m}\boldsymbol{\delta} = \mathbf{T_m}$$

EXAMPLE OF MODAL TRANSFORMATION

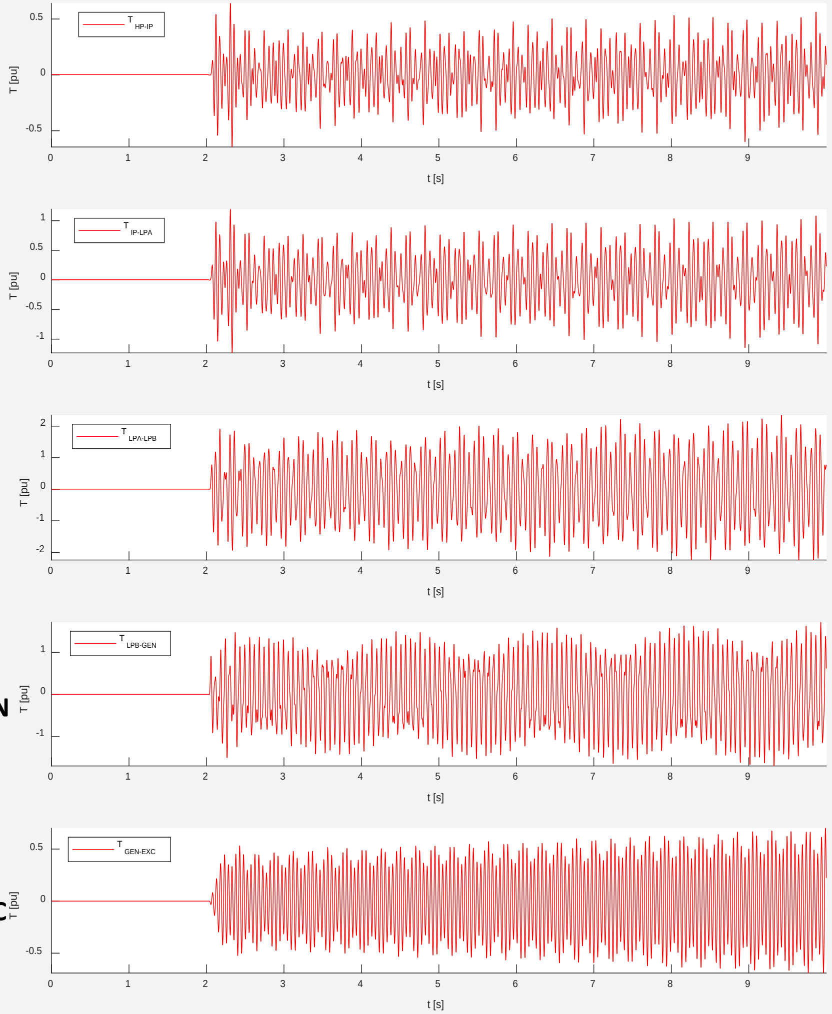
$T_{HP\text{-}IP}$

$T_{IP\text{-}LPA}$

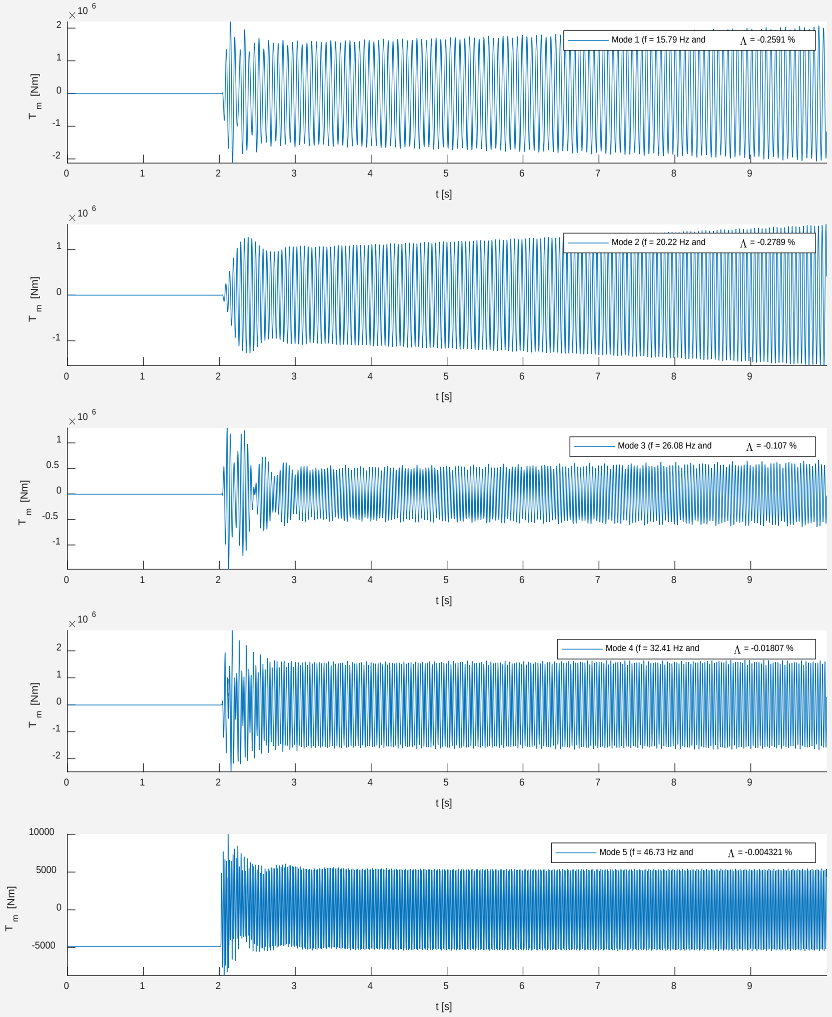
$T_{LPA\text{-}LPB}$

$T_{LPB\text{-}GEN}$

$T_{GEN\text{-}EXC}$



Modal Transformation



Mode1
f=15.79 Hz
Λ=-0.2591 %

Mode2
f=20.22 Hz
Λ=-0.2789 %

Mode3
f=26.08 Hz
Λ=-0.107 %

Mode4
f=32.41 Hz
Λ=-0.01807 %

Mode5
f=46.73 Hz
Λ=-0.004321 %

MATHEMATICAL MODELING OF THE REAL SYSTEM

- Equations of motion :

$$2\mathbf{H}\dot{\boldsymbol{\omega}}(t)=\mathbf{T}(t)-\mathbf{D}\boldsymbol{\omega}(t)-\mathbf{K}\boldsymbol{\theta}(t)$$

$$\dot{\boldsymbol{\theta}}(t)=\boldsymbol{\omega}(t)$$

- Numerical solution using the trapezoidal rule:

$$\frac{\boldsymbol{\omega}(t)-\boldsymbol{\omega}(t-\Delta t)}{\Delta t}=\frac{\mathbf{T}(t)+\mathbf{T}(t-\Delta t)}{2(2\mathbf{H})}-\mathbf{D}\frac{\boldsymbol{\omega}(t)+\boldsymbol{\omega}(t-\Delta t)}{2(2\mathbf{H})}-\mathbf{K}\frac{\boldsymbol{\theta}(t)+\boldsymbol{\theta}(t-\Delta t)}{2(2\mathbf{H})}$$

$$\boldsymbol{\theta}(t)=\frac{\Delta t}{2}\left[\boldsymbol{\omega}(t)+\boldsymbol{\omega}(t-\Delta t)\right]+\boldsymbol{\theta}(t-\Delta t)$$

- Equations to be solved each time-step:

$$\boldsymbol{\omega}(t)=\mathbf{A}^{-1}\left[\mathbf{T}(t)-\mathbf{T}(t-\Delta t)+\boldsymbol{\omega}(t-\Delta t)\mathbf{B}-2\mathbf{K}\boldsymbol{\theta}(t-\Delta t)\right]$$

$$\boldsymbol{\theta}(t)=\frac{\Delta t}{2}\left[\boldsymbol{\omega}(t)+\boldsymbol{\omega}(t-\Delta t)\right]+\boldsymbol{\theta}(t-\Delta t)$$

$$\mathbf{A}=\left[\frac{2}{\Delta t}2\mathbf{H}+\mathbf{D}+\frac{\Delta t}{2}\mathbf{K}\right]$$

$$\mathbf{B}=\left[\frac{2}{\Delta t}2\mathbf{H}-\mathbf{D}-\frac{\Delta t}{2}\mathbf{K}\right]$$

MATHEMATICAL MODELING OF THE MODAL SYSTEM

- Relationships between the coordinate systems:

$$\boldsymbol{\theta}(t) = \mathbf{X}\boldsymbol{\theta}_m(t) \qquad \boldsymbol{\theta}(t) = \mathbf{X}\boldsymbol{\theta}_m(t)$$
$$\dot{\boldsymbol{\theta}}(t) = \mathbf{X}\dot{\boldsymbol{\theta}}_m(t) \quad \Rightarrow \quad \boldsymbol{\omega}(t) = \mathbf{X}\boldsymbol{\omega}_m(t)$$
$$\ddot{\boldsymbol{\theta}}(t) = \mathbf{X}\ddot{\boldsymbol{\theta}}_m(t) \qquad \dot{\boldsymbol{\omega}}(t) = \mathbf{X}\dot{\boldsymbol{\omega}}_m(t)$$

- Transformation of the real system into modal system:

$$\mathbf{T}(t) = 2\mathbf{H}\dot{\boldsymbol{\omega}}(t) + \mathbf{K}\boldsymbol{\theta}(t)$$
$$\mathbf{X}^T\mathbf{T}(t) = 2\mathbf{X}^T\mathbf{H}\mathbf{X}\dot{\boldsymbol{\omega}}_m(t) + \mathbf{X}^T\mathbf{K}\mathbf{X}\boldsymbol{\theta}_m(t) \qquad \mathbf{H}_m = \mathbf{X}^T\mathbf{H}\mathbf{X}$$
$$\mathbf{T}_m(t) = 2\mathbf{H}_m\dot{\boldsymbol{\omega}}_m(t) + \mathbf{K}_m\boldsymbol{\theta}_m(t) \qquad \mathbf{K}_m = \mathbf{X}^T\mathbf{K}\mathbf{X}$$

- Equations to be solved each time-step:

$$\boldsymbol{\omega}_m(t) = \left[\frac{\Delta t}{4\mathbf{H}\mathbf{X}}\right]\left(\mathbf{T}(t) - \mathbf{T}(t-\Delta t) - \mathbf{K}\boldsymbol{\theta}(t) - \mathbf{K}\boldsymbol{\theta}(t-\Delta t)\right) + \boldsymbol{\omega}_m(t-\Delta t)$$
$$\boldsymbol{\theta}_m(t) = \frac{\Delta t}{2}\left[\boldsymbol{\omega}_m(t) + \boldsymbol{\omega}_m(t-\Delta t)\right] + \boldsymbol{\theta}_m(t-\Delta t)$$

IMPLEMENTED MATHEMATICAL FUNCTIONS IN C

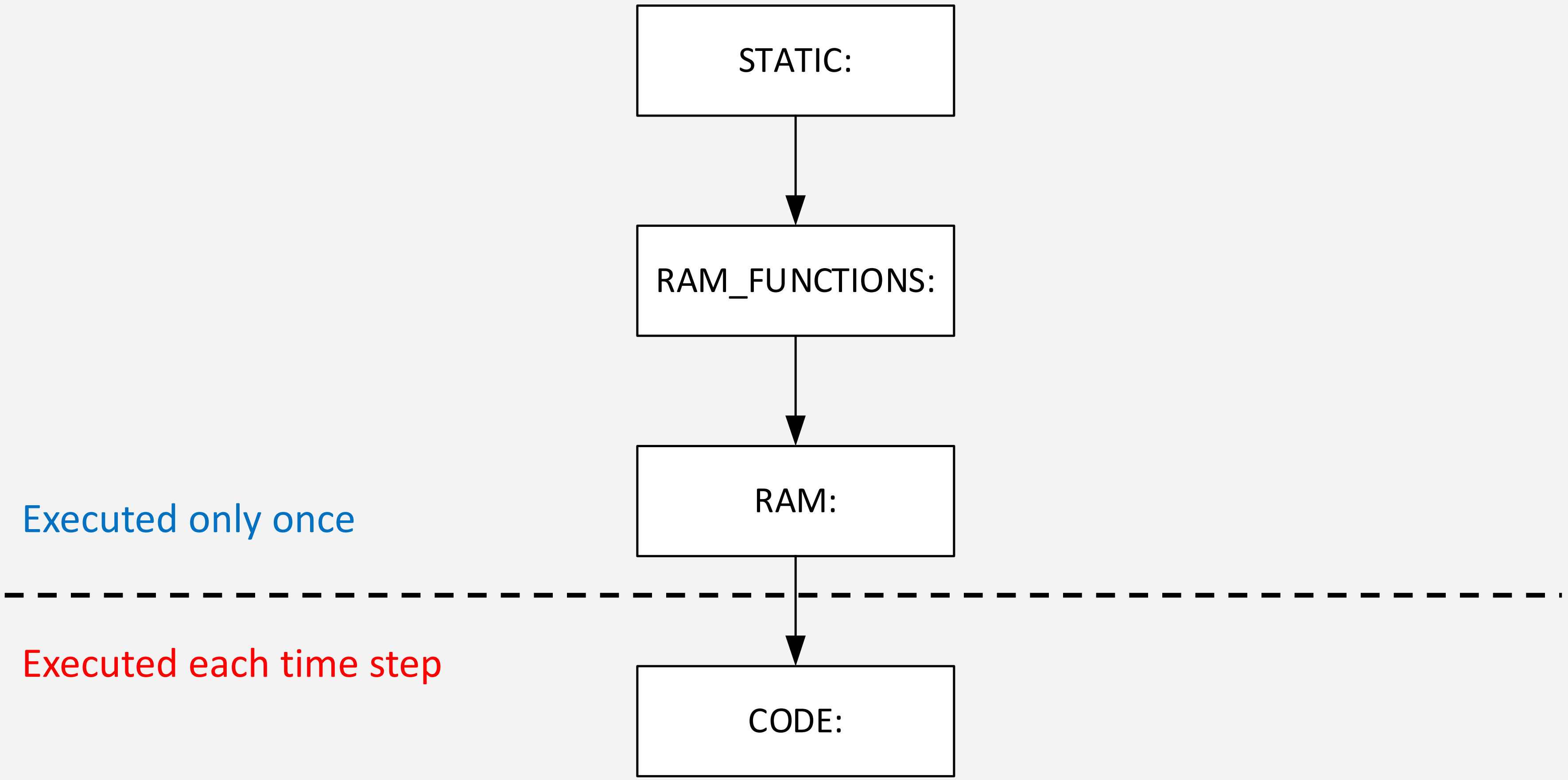
1. **MATINV** - Returns the inverse of a matrix.

2. **MATMULT** – Returns the product of two square matrices and a constant.

3. **MATMULT3** – Returns the product of three square matrices and a constant.

4. **MATEIG** – Calculates the eigenvalues of a given matrix (F). Sorts the eigenvalues in an ascending order and calculates the transformation matrix X and its transpose XT.

RSCAD PROGRAM STRUCTURE



RAM:

Step	C Routines	Mathematical Expressions
1	MATINV(NM, Harr, **iHarr**)	$\mathbf{H}^{-1}$
2	MATMULT(NM, -0.5, iHarr, Karr, **AEIGarr**)	$\mathbf{A}_{EIG}=-0.5*\mathbf{K}*\mathbf{H}^{-1}$
3	MATEIG(NM, AEIGarr, **Xarr**, **XTarr**, **Farr**)	$\mathbf{F}=\text{sqrt}(\text{eig}(\mathbf{A}_{EIG}))$, $\mathbf{X}=\text{sqrt}(F)$, $\mathbf{X}^{T}=\text{transpose}(X)$
4	MATMULT3(NM, 1.0, XTarr, Karr, Xarr, **Kmarr**)	$\mathbf{K}_{M}=\mathbf{X}^{T}*\mathbf{K}*\mathbf{X}$
5	MATMULT3(NM, 2.0, XTarr, Harr, Xarr, **Hmarr**)	$\mathbf{H}_{M}=\mathbf{X}^{T}*2\mathbf{H}*\mathbf{X}$
6	MATMULT(NM, 4.0, Harr, Xarr, **FHXarr**)	$\mathbf{FHX}=4*\mathbf{H}*\mathbf{X}$
7	MATINV(NM, Xarr, **iXarr**)	X^{-1}

NM – Number of masses

CODE:

Step 1: Calculate Initial Values (Only once)

$$\boldsymbol{\theta}(0) = f(\mathbf{T}(0)),\ \boldsymbol{\theta}_m(0) = \mathbf{X}^{-1}\boldsymbol{\theta}(0)$$

Step 2: Calculate $\omega(t)$

$$\boldsymbol{\omega}(t) = \mathbf{A}^{-1}\left[\mathbf{T}(t) - \mathbf{T}(t-\Delta t) + \boldsymbol{\omega}(t-\Delta t)\mathbf{B} - 2\mathbf{K}\boldsymbol{\theta}(t-\Delta t)\right]$$

Step 3: Calculate $\theta(t)$

$$\boldsymbol{\theta}(t) = \frac{\Delta t}{2}\left[\boldsymbol{\omega}(t) + \boldsymbol{\omega}(t-\Delta t)\right] + \boldsymbol{\theta}(t-\Delta t)$$

Step 4: Calculate $\omega_m(t)$

$$\boldsymbol{\omega}_m(t) = \left[\frac{\Delta t}{4\mathbf{HX}}\right]\left(\mathbf{T}(t) - \mathbf{T}(t-\Delta t) - \mathbf{K}\boldsymbol{\theta}(t) - \mathbf{K}\boldsymbol{\theta}(t-\Delta t)\right) + \boldsymbol{\omega}_m(t-\Delta t)$$

Step 5: Add Modal Damping D_m

$$\boldsymbol{\omega}_m(t) = \boldsymbol{\omega}_m(t)\cdot\left[1 - D_m(t)\right]$$

Step 6: Calculate $\theta_m(t)$

$$\boldsymbol{\theta}_m(t) = \frac{\Delta t}{2}\left[\boldsymbol{\omega}_m(t) + \boldsymbol{\omega}_m(t-\Delta t)\right] + \boldsymbol{\theta}_m(t-\Delta t)$$

Step 7: Inverse Transformation of $\omega_m(t)$

$$\boldsymbol{\omega}(t) = \mathbf{X}\boldsymbol{\omega}_m(t)$$

Step 8: Inverse Transformation of $\theta_m(t)$

$$\boldsymbol{\theta}(t) = \mathbf{X}\boldsymbol{\theta}_m(t)$$

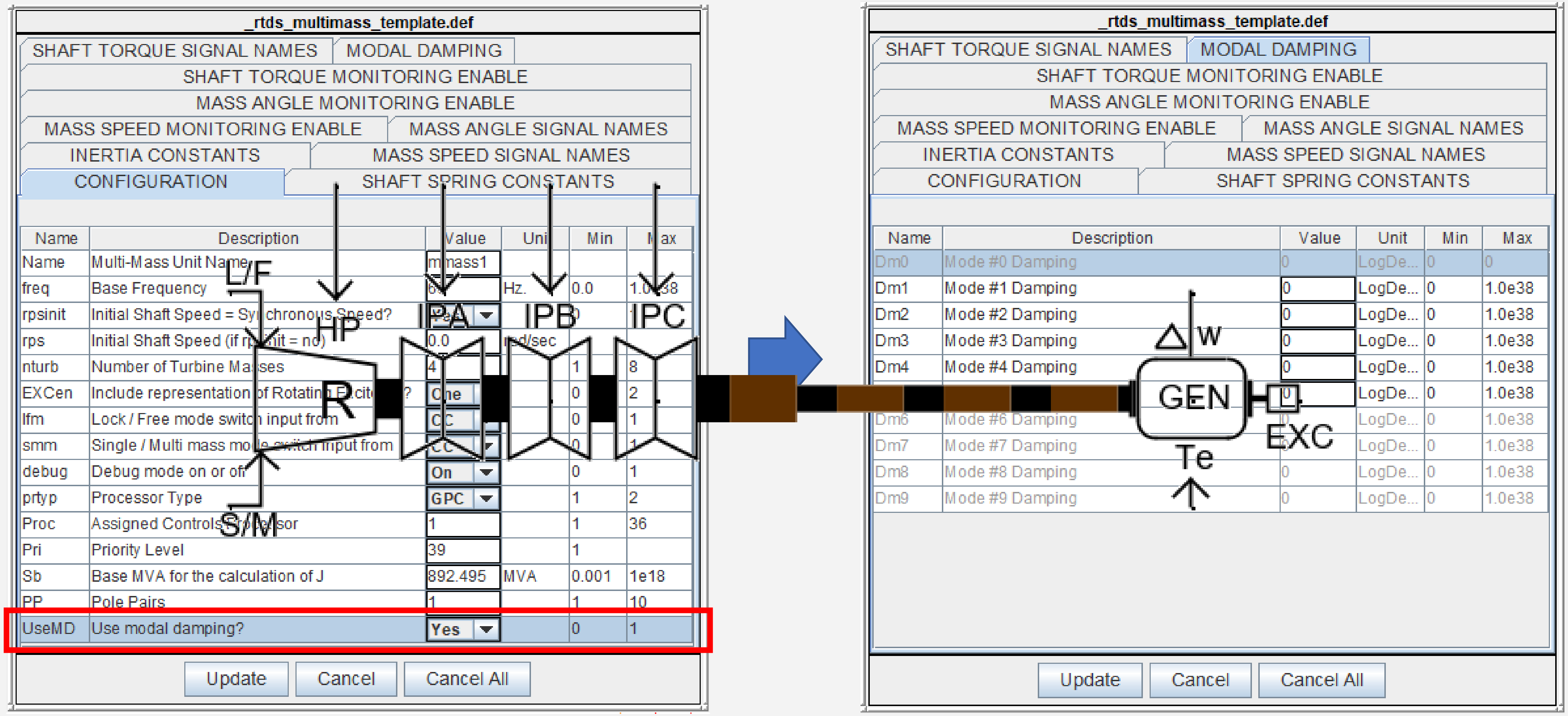
Step 9: Write Output

$$\boldsymbol{\theta}(t),\ \boldsymbol{\theta}_m(t),\ \boldsymbol{\omega}(t),\ \boldsymbol{\omega}_m(t)$$

Step 10: Update History Terms

$$\boldsymbol{\theta}(t-\Delta t) = \boldsymbol{\theta}(t),\ \boldsymbol{\theta}_m(t-\Delta t) = \boldsymbol{\theta}_m(t),\ \boldsymbol{\omega}(t-\Delta t) = \boldsymbol{\omega}(t)\ \ldots$$

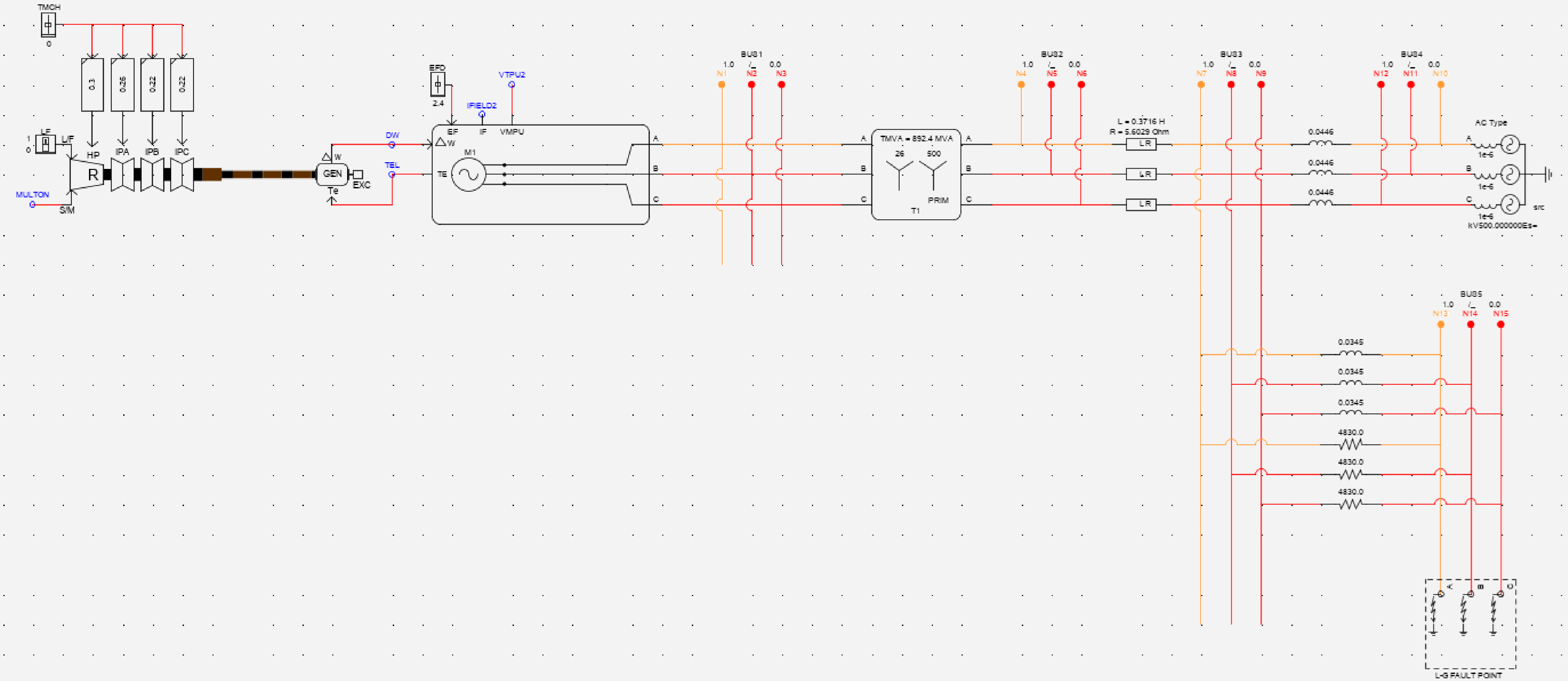
IMPLEMENTATION OF THE MODEL



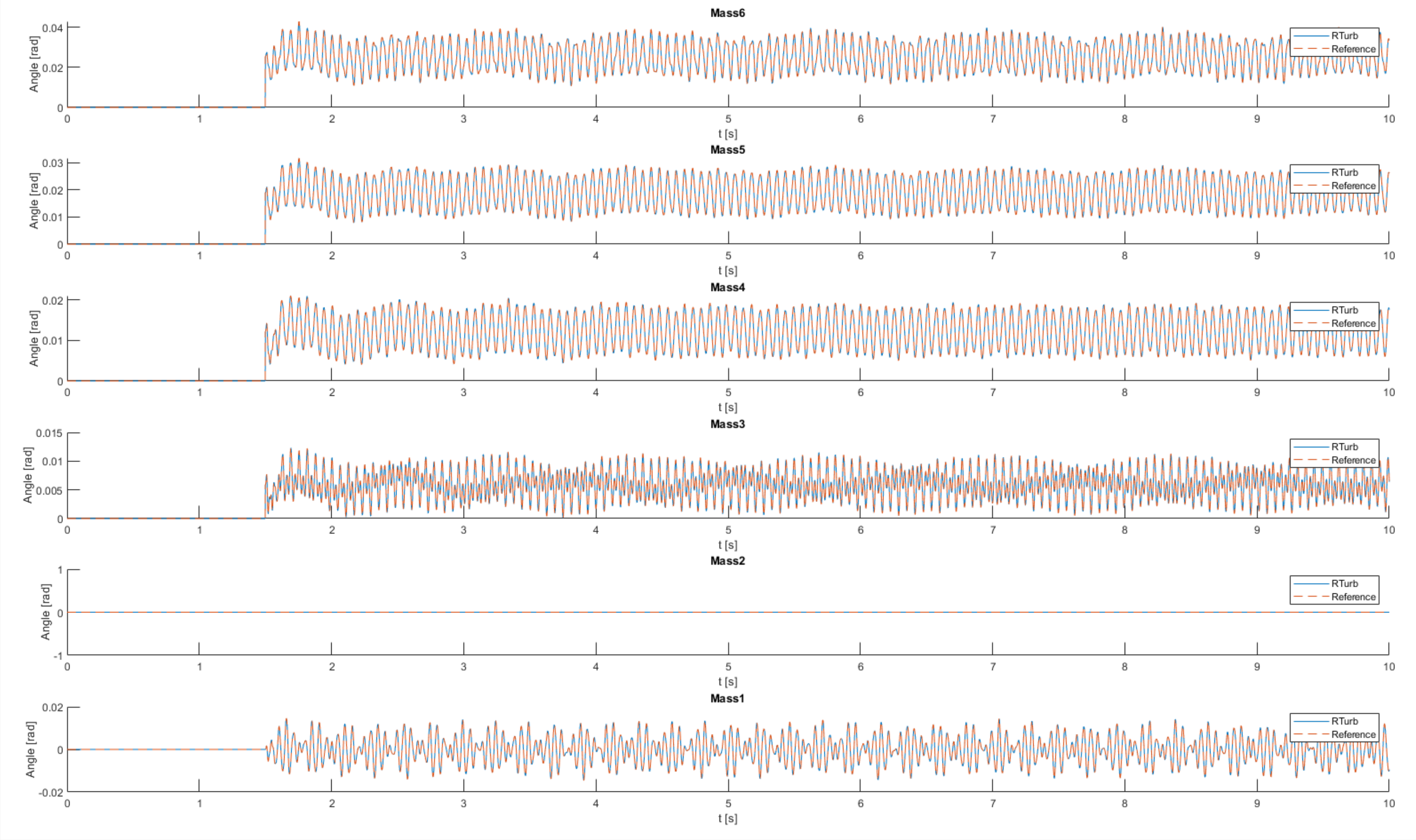
Name	Description	Value	Unit	Min	Max
Name	Multi-Mass Unit Name	mmass1			
freq	Base Frequency		Hz.	0.0	1.0e38
rpsinit	Initial Shaft Speed = Synchronous Speed?				
rps	Initial Shaft Speed (if rpsinit = no)	0.0	rad/sec		
nturb	Number of Turbine Masses	4		1	8
EXCen	Include representation of Rotating Exciter?	One		0	2
lfm	Lock / Free mode switch input from	CC		0	1
smm	Single / Multi mass mode switch input from	CC		0	1
debug	Debug mode on or off	On		0	1
prtyp	Processor Type	GPC		1	2
Proc	Assigned Controls Processor	1		1	36
Pri	Priority Level	39		1	
Sb	Base MVA for the calculation of J	892.495	MVA	0.001	1e18
PP	Pole Pairs	1		1	10
UseMD	Use modal damping?	Yes		0	1

Name	Description	Value	Unit	Min	Max
Dm0	Mode #0 Damping	0	LogDe...	0	0
Dm1	Mode #1 Damping	0	LogDe...	0	1.0e38
Dm2	Mode #2 Damping	0	LogDe...	0	1.0e38
Dm3	Mode #3 Damping	0	LogDe...	0	1.0e38
Dm4	Mode #4 Damping	0	LogDe...	0	1.0e38
[illegible]	[illegible]	0	LogDe...	0	1.0e38
Dm6	Mode #6 Damping	0	LogDe...	0	1.0e38
Dm7	Mode #7 Damping	0	LogDe...	0	1.0e38
Dm8	Mode #8 Damping	0	LogDe...	0	1.0e38
Dm9	Mode #9 Damping	0	LogDe...	0	1.0e38

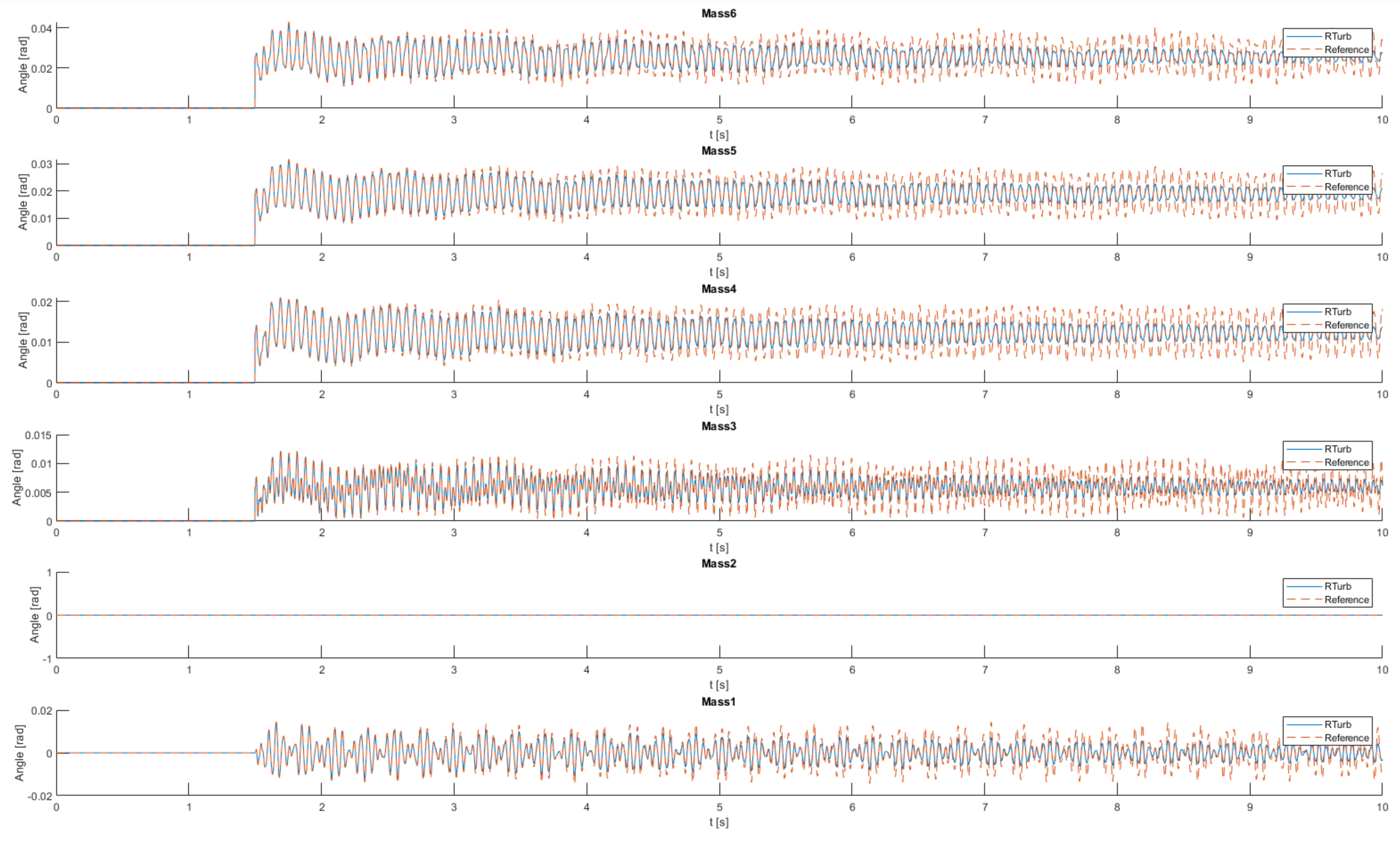
THE FIRST BENCHMARK MODEL



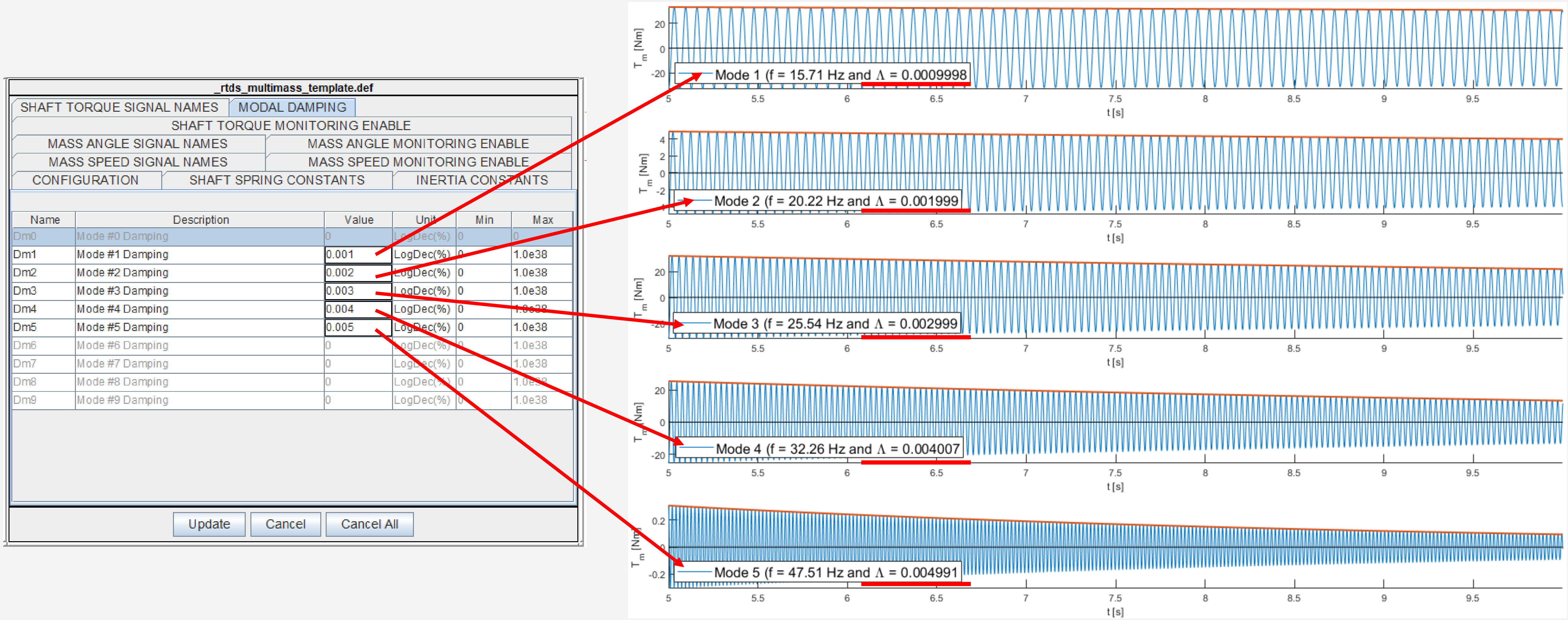
COMPARISON WITH RSCAD (DM=0)



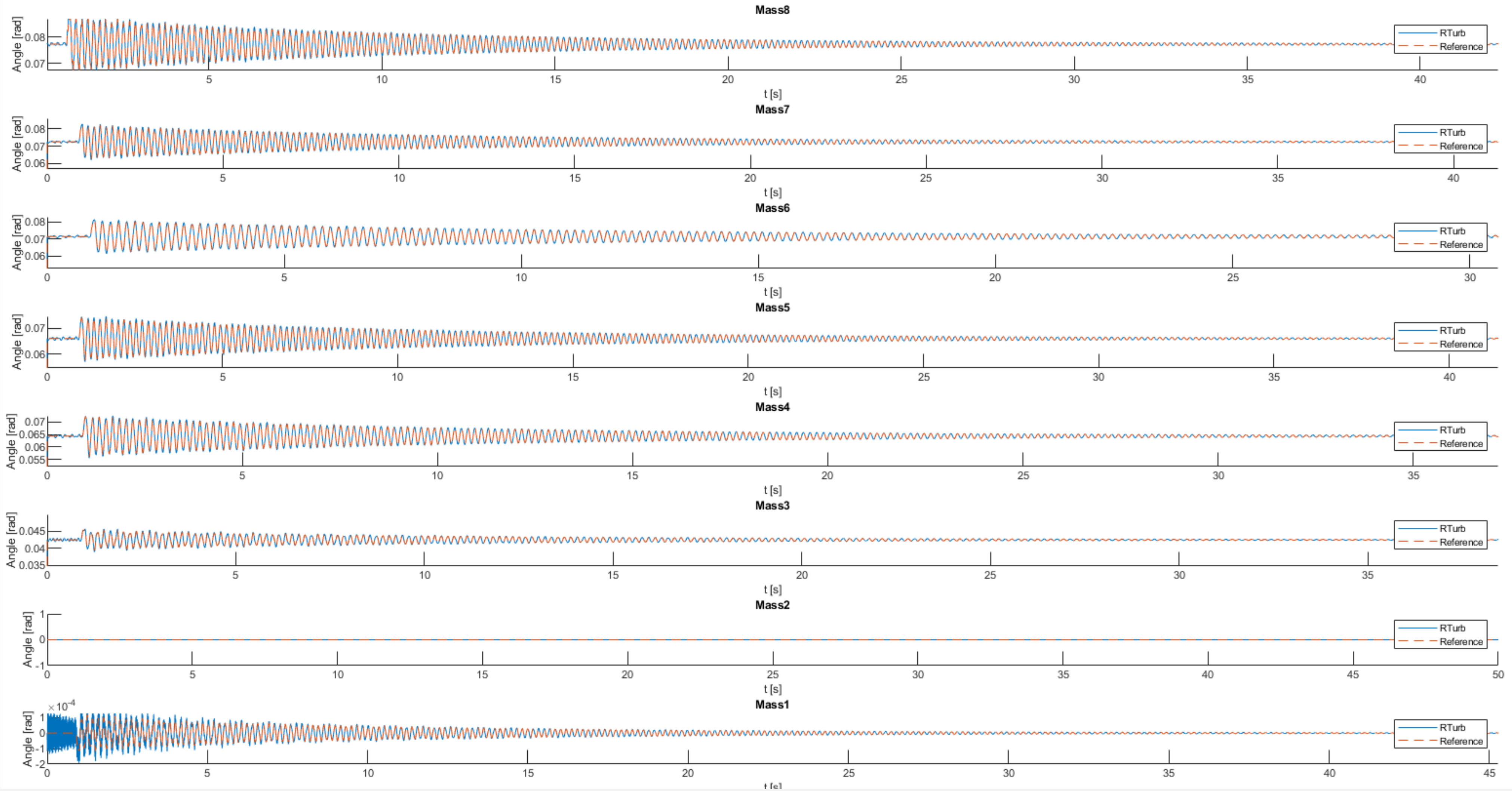
COMPARISON WITH RSCAD (DM>0)



MODAL TORQUES (DM > 0)



COMPARISON WITH PSS®NETOMAC



CONCLUSION

- An enhanced multi-mass model was developed
- Validation against the classical RSCAD model
- Validation as a standalone model
- Validation against the PSS®NETOMAC model