An Efficient Saturation Algorithm for Real Time Synchronous Machine Models using Flux Linkages as State Variables

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***Abstract* - This paper presents an efficient method of calculation concerning D-axis saturation in a synchronous machine model when winding flux linkages are the state variables.**

A method is presented for calculation of a saturation indicator S based on a linear combination of the flux linkages in the D-axis windings. A method is then presented for developing a look-up curve F(S) which is dependent on the saturation indicator. The product of S times F(S) produces ψ_{md}, the main mutual magnetizing flux linkage for the D-axis. The component of flux linkage due to leakage in each winding on the D-axis can be obtained by subtracting ψ_{md} from the flux linkage of the particular winding. D-axis winding currents are subsequently obtained by multiplying the components of winding flux linkages due to leakage by the pre-calculated inverse of the leakage inductance matrix.

I. Introduction

THERE are various synchronous machine models, based on Park's equations, which use either winding currents or flux linkages on the D and Q axes as state variables [1].

These models generally receive node voltages from an AC system which are transformed into D-axis and Q-axis voltages. The D-axis and Q-axis voltages, the field voltage, the cross-generation voltages and the resistance drop voltages are integrated either directly to obtain D and Q axis flux linkages or using an inverse incremental inductance matrix to obtain D and Q axis currents. When fluxes are used as state variables, it is still necessary to be able to determine the winding currents having consideration for saturation of the machine. The models use the D and Q axis stator winding currents to produce injection currents for the AC system and other necessary information.

When using winding currents as state variables, the magnetizing current on an axis i_{md} is obtained simply by summing the per unit currents in all of the windings on the axis. Subsequently, the main mutual magnetizing flux linkage ψ_{md} can be obtained from a look-up curve using magnetizing current i_{md} as the input argument.

This paper presents a similarly efficient method for handling saturation on the D-axis of a real-time synchronous machine model when winding flux linkages are the state variables rather than currents. The described method comprises a technique for expressing the winding currents based on winding flux linkages. This technique is required in order to be able to express voltage drops, torque, powers and the injections which are based on winding currents.

Specifically, a method is presented for calculating a saturation indicator S that is a linear combination of winding flux linkages on the D-axis according to constant coefficients. A method is then presented for developing a look-up curve F(S) which is dependent on the saturation indicator. The product of S times F(S) for a given S provides the main mutual magnetizing flux linkage ψ_{md} for the D-axis. The flux linkages for the windings each minus the main mutual flux linkage ψ_{md} provide the flux linkages in each particular winding due to leakage. Winding currents are obtained by multiplying the flux linkages due to leakage by a pre-calculated inverse leakage inductance matrix.

The method has been implemented in a synchronous machine model for calculation in the RTDS real time simulator hardware. The model is described and brief test results are presented.

II. Background

This paper considers the case in which winding fluxes are the state variables for integration. For this case, new fluxes are calculated in each time-step based on the integration of the state equation shown in Equation 1.

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$$\frac{d}{dt}\begin{bmatrix}\Psi_d\\ \Psi_{fd}\\ \Psi_{kd}\\ \Psi_q\\ \Psi_{fq}\\ \Psi_{kq}\end{bmatrix} = \begin{bmatrix}V_d\\ V_f\\ 0\\ V_q\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}\cdot & \cdot & \cdot & -v & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ v & \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\end{bmatrix}\begin{bmatrix}\Psi_d\\ \Psi_{fd}\\ \Psi_{kd}\\ \Psi_q\\ \Psi_{fq}\\ \Psi_{kq}\end{bmatrix} - \begin{bmatrix} & & \\ & R & \\ & & \end{bmatrix}\begin{bmatrix}i_d\\ i_{fd}\\ i_{kd}\\ i_q\\ i_{fq}\\ i_{kq}\end{bmatrix} \quad (1)$$

The magnetic circuit on the D-axis of a machine is often modeled according to the illustration in Figure 1. The reactances and currents defined in Figure 1 are used in the discussion below.

The matrix equation relating D-axis fluxes to D-axis currents is as shown in Equation 2.

$$\begin{bmatrix}\Psi_d\\ \Psi_{fd}\\ \Psi_{kd}\end{bmatrix} = \begin{bmatrix}X_{md}+X_a & X_{md} & X_{md}\\ X_{md} & X_{md}+X_{kf}+X_{fd} & X_{md}+X_{kf}\\ X_{md} & X_{md}+X_{kf} & X_{md}+X_{kf}+X_{kd}\end{bmatrix}\begin{bmatrix}i_d\\ i_{fd}\\ i_{kd}\end{bmatrix} \quad (2)$$

Review of machine modeling methods indicates that X_{md} in Equation 2 is usually expressed as saturating according to the main magnetizing current for the axis, $i_{md} = i_d + i_{fd} + i_{kd}$. Therefore, X_{md} can be expressed as being a function of magnetizing current, $X_{md}(i_{md})$. This presents a difficulty in that the inverse of the matrix in Equation 2 (required to find the currents i_d, i_{fd}, and i_{kd}.) is itself a function of the currents i_d, i_{fd}, and i_{kd}.

This paper presents a simple method for efficiently and accurately finding i_d, i_{fd}, and i_{kd} based on a pre-calculated and stored look-up curve F(S) for any set of flux linkages on the D-axis: ψ_d, ψ_{fd}, and ψ_{kd}.

III. Expressing The Main D-Axis Mutual Magnetizing Flux As A Function Of D-Axis Fluxes

The unique components of the matrix in Equation 2 can be defined as follows:

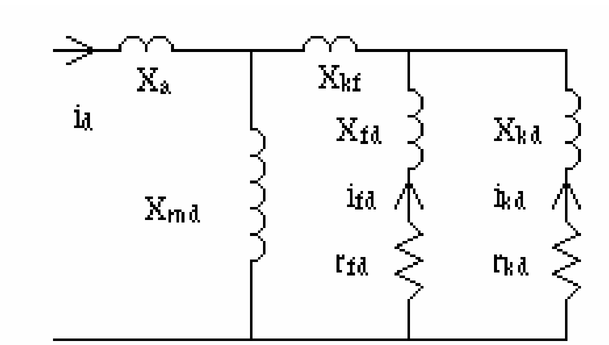


Figure 1

$M = X_{md}$ $\quad A = X_a$

$B = X_{kf} + X_{fd}$ $\quad C = X_{kf}$

$D = X_{kf}$ $\quad E = X_{kf} + X_{kd}$

Of course, the component M is the saturated main mutual magnetizing inductance for the D-axis which relates ψ_{md} to i_{md} according to $\psi_{md} = M(i_{md}) * i_{md}$.

Based on the above definitions, the matrix in Equation 2 can be expressed as:

$$L = \begin{bmatrix}M+A & M & M\\ M & M+B & M+C\\ M & M+D & M+E\end{bmatrix} \quad (3)$$

If the inverse of L were known, we could express the main magnetizing current as:

$$i_{md} = \begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix} & L^{-1} & \end{bmatrix}\begin{bmatrix}\Psi_d\\ \Psi_{fd}\\ \Psi_{kd}\end{bmatrix} \quad (4)$$

Fortunately, the inverse of L can be expressed in a simple form using Cramer's rule. Of course, the expression includes the variable M as well as the constants, A, B, C, D, and E.

We may further choose to define fixed constants:

$K_1 = B * E - C * D = X_{kf} * [X_{fd} + X_{kd}] + X_{fd} * X_{kd}$

$K_2 = A * E - A * D = X_a * X_{kd}$

$K_3 = A * B - A * C = X_a * X_{fd}$

and $K_4 = A * K_1$

Using the inverse of L and the above definitions, Equation 4 can be re-written as Equation 5.

$$i_{md} = \frac{K_1 \Psi_d + K_2 \Psi_{fd} + K_3 \Psi_{kd}}{M(K_1 + K_2 + K_3) + K_4} \quad (5)$$

The denominator of the right hand side of Equation 5 is the determinant of matrix L. M is not constant and has yet to be expressed other than as a function of i_{md}. We choose to define the numerator of the right hand side of Equation 5 to be a saturation indicator, S. Since K_1, K_2, K_3 and K_4 are known positive constants, the saturation indicator S can be efficiently calculated for any new set of D-axis fluxes.

Multiplying both sides of Equation 5 by the determinant and substituting with S yields:

$$(K_1 + K_2 + K_3)\, M\, i_{md} + K_4\, i_{md} = S \quad (6)$$

As noted above, M * i_{md} is equal to ψ_{md}, the main mutual flux linkage for the D-axis.

Therefore, Equation 6 can be re-written as

$$\psi_{md} * (K_1 + K_2 + K_3) + K_4 * i_{md} = S \quad (7)$$

The manufacturer of the synchronous machine typically provides saturation data sufficient to generate a saturation curve of main magnetizing flux linkage versus magnetizing current, $\psi_{md}(i_{md})$. Given $\psi_{md}(i_{md})$, it is possible to describe the saturation indicator S as a function of i_{md} based on Equation 7. If the simple and realistic assumption is made that the $\psi_{md}(i_{md})$ curve is of constant or decreasing positive slope with increasing i_{md}, then the function $S(i_{md})$ will similarly have constant or decreasing positive slope with increasing i_{md}. Consequently, there will be a unique value of S for every value of i_{md} according to the monotonically increasing curve $S(i_{md})$. Conversely, for every value of S there will be a unique value of i_{md}. Consequently, any function of i_{md} can be converted into a function of S. Therefore, the curve $M(i_{md})$ can be converted to a curve of M(S).

If Equation 5 is multiplied by M and S is substituted into the equation, then we can express ψ_{md} as follows:

$$\Psi_{md} = \frac{M}{M(K_1 + K_2 + K_3) + K_4}\, S \quad (8)$$

According to the above discussion, it is convenient to define a look-up curve based on S as follows:

$$F(S) = \frac{M}{M(K_1 + K_2 + K_3) + K_4} \quad (9)$$

The curve defined in Equation 9 can be pre-calculated and stored. Based on Equation 9, Equation 8 can be re-written as:

$$\psi_{md}(S) = F(S) * S \quad (10)$$

IV. UTILIZING ψ_{md} WITH ψ_d, ψ_{fd}, AND ψ_{kd} TO PRODUCE D-AXIS CURRENTS

Considering that the magnetizing current on the D-axis can be expressed as

$$i_{md} = i_d + i_{fd} + i_{kd} \text{ and that} \quad (11)$$

$$\psi_{md} = X_{md} * i_{md} \quad (12)$$

Equation 2 can be separated and re-written as:

$$\begin{bmatrix} \Psi_d \\ \Psi_{fd} \\ \Psi_{kd} \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ 0 & B & C \\ 0 & D & E \end{bmatrix} \begin{bmatrix} i_d \\ i_{fd} \\ i_{kd} \end{bmatrix} + \begin{bmatrix} \Psi_{md} \\ \Psi_{md} \\ \Psi_{md} \end{bmatrix} \quad (13)$$

The inverse of the constant leakage inductance matrix in Equation 13 exists and is easily pre-calculated off-line in advance of the real-time simulation. Subsequently, during real-time simulation, winding currents can be calculated for the D-axis according to:

$$\begin{bmatrix} i_d \\ i_{fd} \\ i_{kd} \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ 0 & B & C \\ 0 & D & E \end{bmatrix}^{-1} \begin{bmatrix} \Psi_d - \Psi_{md} \\ \Psi_{fd} - \Psi_{md} \\ \Psi_{kd} - \Psi_{md} \end{bmatrix} \quad (14)$$

V. SUMMARY OF THE REAL TIME METHOD FOR HANDLING SATURATION

The procedure is as follows:

1. Calculate the saturation indicator S, based on the new values of winding fluxes ψ_d, ψ_{fd}, and ψ_{kd} produced by integration of the state equation 1. Calculation of S is according to:

$$S = K_1 * \psi_d + K_2 * \psi_{fd} + K_3 * \psi_{kd} \quad (15)$$

2. Calculate ψ_{md} = F(S) * S where F(S) is the pre-calculated look-up curve.

3. Calculate winding currents based on Equation 14.

VI. TEST RESULTS

The machine model implemented in RTDS appears as shown in the following Figure 2. The generator model is solved on one processor and optionally includes the generator transformer and generator bus as shown.

At the start-up of a real-time simulation, the machine model is initialized according to the User specified initial terminal conditions of the machine. These conditions are available from a load flow for larger systems. The terminal conditions include initial terminal voltage magnitude, initial angle of the A phase terminal voltage sine wave, and the initial real and reactive power out of the machine.

Based on the initial terminal conditions, an off-line calculation of the initial D and Q-axis winding currents is made by the RTDS compiler prior to the commencement of the real-time simulation. This includes a calculation of the required initial field current. The initial winding flux linkages are available, based upon the winding currents. These off-line calculations take into consideration the saturation characteristic of the machine and the load. The saturation characteristic for the RTDS machine can be entered according to saturation factors SE(1.0) and SE(1.2) or according to a curve entered as points.

The validity of the real-time saturation calculations is confirmed by the fact that the machine operates during real-time simulation according to the specified initial conditions. If saturation were calculated improperly during real-time simulation, then the specified terminal voltage would not be maintained for a pre-calculated field current predicted off-line.

The simple test circuit in Figure 2 was used for verifications during the development of the machine model. The rated line-to-line RMS voltage of the generator, transformer primary and secondary, and the source are 8.66025 kV. The rating of the transformer and the generator are both 75 MVA. The total reactance of the source and the transformer is Xl = 0.1 per unit. The specified initial value of the ideal source is 0.9512885 per unit voltage at 30 degrees (to compensate for the 30 degree delta lag in the transformer). The specified initial voltage of the machine is 1.0 per unit at 3.013 degrees. With these settings, the load flow calculation for the network indicates that the machine will need to supply 37.5 MW and 37.5 MVAR out of the machine toward the source. Therefore, these terminal power values are entered into the machine as the initial P and Q values along with the initial terminal voltage described above. When the case is compiled using the RTDS compiler, the compiler uses the terminal conditions and the machine parameters including the saturation curve to pre-calculate all initial winding currents and the initial rotor position. The initial field current for the specified conditions is calculated as a normalized value of 2.1754195. A corresponding fixed value of normalized field voltage is applied to the field with a slider prior to the start of the test case as shown in Figure 2. When the case is run, the machine terminal voltage rises to 1.00024 per unit with power out of the machine of 37.495 MW and 37.5126 MVAR. This closely matches the expected load flow condition of 1.0 per unit voltage with 37.5 MW and 37.5 MVAR.

This simple test demonstrates that saturation is being calculated correctly during real-time simulation by confirming that off-line calculations and real-time calculations produce results that are in agreement.

VII. Summary

This paper provides an efficient and accurate method for converting D-axis winding fluxes to D-axis winding currents during real-time simulation for a machine model that saturates on the D-axis.

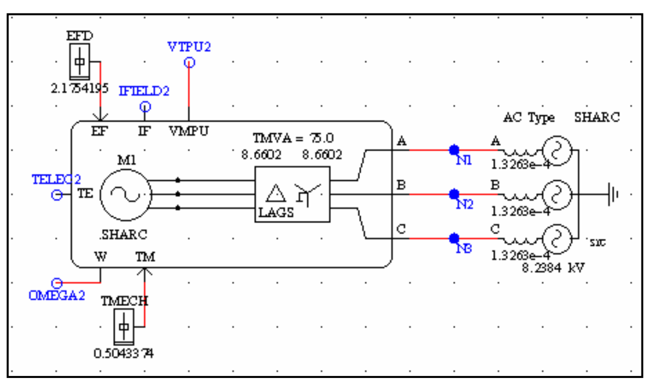
In real-time simulation, the efficiency of this method is important because of the need to keep the simulation time-step as short as possible. However, there are other benefits. In particular, the currents in the state equation (1) can be eliminated in favour of the state variable fluxes. This eliminates a potential 1 time-step delay in applying resistance drop voltages caused by winding currents.

The machine model has also been tested using open circuit, short circuit and load test methods. The results produced by the machine also conform with the Sub-synchronous Resonance Benchmark Test[2].

The saturation method has been extended to support saturation in an induction machine model which includes saturation on both the D and Q axes.

VIII. References

1. D.A. Woodford, A.M. Gole, and R.W. Menzies, “Digital Simulation of DC Links and AC Machines”, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-102, No. 6, June 1983, p. 1616-1623.
2. IEEE Subsynchronous Resonance Task Force, “First



Benchmark Model for Computer Simulation of Subsynchronous Resonance", IEEE Transactions on PAS, Vol. PAS-96, no. 5, September/October 1977.

Periodicals:

[1] D.A. Woodford, A.M. Gole, and R.W. Menzies, "Digital Simulation of DC Links and AC Machines", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-102, No. 6, June 1983, p. 1616-1623.
[2] IEEE Subsynchronous Resonance Task Force, "First Benchmark Model for Computer Simulation of Subsynchronous Resonance", IEEE Transactions on PAS, Vol. PAS-96, no. 5, September/October 1977.